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THE INFLUENCE OF STRUCTURAL AND KINEMATIC PARAMETERS OF WORKING BODIES OF THE MEAT GRINDERS ON ITS PRODUCTIVITY

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Abstract: Meat grinding tops are used in technological lines of production of sausages, canned meat and stuffed semi-finished products. An increase in the specific productivity of tops is economically sound which allows the consumer to reduce significantly the capital and operational costs. It has been established by the authors that not all the area of grating is supplied at any time with meat raw materials by means of the top screw, but only a certain sector is, which significantly reduces the productivity of the top. The researches have been performed using the mathematical modeling of the process of supply of meat raw materials in the top with the subsequent check of adequacy of mathematical model by means of natural experiments under production conditions. As a result, a mathematical model of process of supply of raw materials in the top has been developed that allows to increase significantly the accuracy of determination of its productivity and also to set the most rational structural and kinematic parameters of the working bodies of the top. It has been established that most of the volume of raw materials is supplied by the screw through the grinding knot. The presence of gratings and knives in the grinding knot and their geometrical parameters provide the formation of reverse flows of raw materials through a gap between the screw and the internal surface of the working cylinder and along the screw channel of the screw. The following optimum technological parameters of the process of supply of raw materials in the top have been determined: the outer diameter of the output grating is 0.15–0.155 m; the rotation frequency of the screw is 4.5–5.2 sec^{-1}; the angle of lead of rounds of the screw is 4.8–5.5 degrees; the area of frontal projection of a knife blade is 0.001–0.0011 m^2; the thickness of the output grating is 0.0075–0.0082 m.

Keywords: Meat grinder, productivity, mathematical model, screw, grinding set

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INTRODUCTION

Meat grinding tops are used in technological lines of production of sausages, canned meat and stuffed semi-finished products. They are characterized by quite a simple design, safety in operation and low energy consumption. The most important technical indicator of tops is their productivity which defines a possibility of use of the machine as part of the designed processing line. The increase in productivity in practice is reached, most often, by an increase in the geometry of working bodies - gratings, knives and the screw. However, an increase in the specific productivity of tops is economically sound which allows the consumer to reduce significantly the capital and operational costs. Today the customary ways of increase in specific productivity (an increase in the rotation frequency of the screw, the application of gratings with larger holes, the use of trimmers) mostly exhausted their potential and do not allow to improve significantly the technical and economic indices of the machine.

An approach applied by most of researchers [1] when determining the productivity of the top is of interest in the context of this problem. According to it, the productivity of the top is determined by the formula $G = f \quad S_E$ where S_E is screw efficiency (S_E = 0.25–0.35). Such a low value of coefficient is explained [1] by the loss of raw materials through the gaps between the screw and a wall of the working cylinder, by slipping of raw materials during the rotation of the screw and so forth. However, in our opinion, the loss of about 70% of the greatest possible theoretical productivity is an excessively great value for the factors specified by the researchers.

The results of the authors' researches [2] demonstrate the existence of the following features when supplying meat raw materials by means of the top of the screw - not all the area of grating is supplied at any time with meat raw materials by means of the top screw, but only a certain sector is which is measured from the end of a round of the screw. It makes the productivity of the top twice as low [3]. In our opinion, this effect is the one that causes such a low value of screw efficiency $S_E = 0.2–0.35$ during the operation of the screw. The development of theoretical provisions and mathematical apparatus technique which would allow to determine the productivity of the top more precisely is urgent, considering all the necessary structural and kinematic parameters of the machine, and also the rheological parameters of raw materials.

Some approaches to the determination of productivity of the top are known. A number of researchers [1] suggest to determine the productivity of the top using the supplying ability of the screw or the grinding ability of the grinding knot. However these approaches cause considerable errors when making a calculation. The approach given in the works [4, 5] allows for more exact results. However it does not allow to display the above stated heterogeneity of supplying the work surface of gratings with raw materials. The approach offered by V. N. Potokin and N. A. Vishelevskiy offers to determine the productivity of the top when counting the pressure of supply of raw materials in the grinding knot of the top. The rheological properties of meat raw materials are described by V. V. Goryachev in the approach based on the provisions of the investigative theory of viscoelasticity. However such a technique of determination of productivity of the top does not allow to display the structural and kinematic parameters of the screw which essentially prevents it from being used in practice.

The purpose of the study is the development of a mathematical model of the process of supply of raw materials in the top which would allow to increase significantly the accuracy of determination of its productivity and also to set the most rational structural and kinematic parameters of the working bodies of the top.

OBJECTS AND METHODS OF STUDY

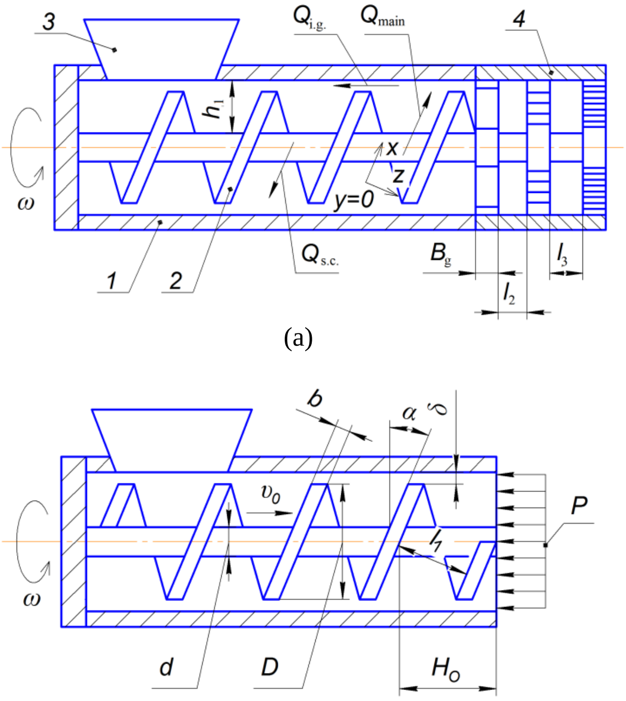
The researches were conducted using the mathematical modeling of the process of supply of meat raw materials in the top with the subsequent check of adequacy of mathematical model by means of natural experiments under production conditions.

The movement of raw materials in the working cylinder can be described by means of provisions of mechanics of continuous medium. At the same time the specific effect which consists in forcing raw materials only in a certain sector of cross section of the screw is expedient to be considered by means of the productivity coefficient K_Q (the physical sense of productivity coefficient K_Q is revealed in more detail in [6]). Then the productivity of the top is defined by the statement:

$$G = K_Q (Q_{main} - Q_{i.g.} - Q_{s.c.})\rho, \qquad (1)$$

where Q_{main} is the main flow of raw materials through the grinding knot, m^3/s; $Q_{i.g.}$ is a reverse flow of raw materials through the gap between the external surface of rounds of the screw and the internal surface of the working cylinder, m^3/s; $Q_{s.c.}$ is a reverse flow of raw materials along the screw channel of the screw, m^3/s; ρ is the density of raw materials, kg/m^3.

Fig. 1 provides the design scheme of the top. It consists of the working cylinder 1 in which the screw 2 is set. Raw materials are supplied to the working cylinder from the bunker 3, and go out of the working cylinder through the grinding knot 4 in which they are ground.



(a)

(b)

Fig. 1. Design scheme of the mathematical model of the process of supply of raw materials in the top: (a) the structure of the top; (b) the model of the working area.

The productivity of the top is defined, first of all, by the supplying ability of the screw which forms the main flow of raw materials Q_{main} through the grinding knot. In the absence of the grinding knot the value of productivity of the top would be maximum at the output of the working cylinder (we consider that the working screw of the top is supplied with raw materials from the bunker constantly, the holes of gratings of the grinding set are not clogged with particles of raw materials, the gratings and knives are pointed properly, etc.). However the following major factors provide the reduction of productivity.

The presence of gratings and knives in the grinding knot and their corresponding geometrical parameters provide the formation of hydraulic resistance of the grinding knot P, which prevents from the free effluence of raw materials from the working cylinder under the influence of the pressure produced by the screw. Thereof the reverse flows which characterize losses of productivity are formed: through the gap between the external surface of rounds of the screw and the internal surface of the working cylinder $Q_{i.g.}$; along the screw channel of the screw between its rounds $Q_{s.c.}$.

The mathematical description of movement of continuous medium (in this case – meat), can be obtained by the solution of system of the equations which consist of a continuity equation, a motion equation, an energy equation and equations of rheological condition of raw materials. At the same time, to obtain a rigorous solution of such a system of equations in case of supplying the curvilinear channel of the screw with real (with a rather complex set of properties) raw materials is a rather difficult task. However it can be simplified by adding certain assumptions and restrictions.

The rotating screw and the stationary working cylinder are replaced with a fixed screw and a rotating working cylinder. The screw channel which is formed by the space between screw rounds is straightened. That is instead of the rectangular-sectioned screw channel a straightened pipe the length of which is equal to the length of the screw channel is put, and which has a movable (according to the preliminary assumption) upper surface. The system of rectangular coordinates is chosen according to Fig. 1: The *x*-axis is along the channel, the axis *y*-axis is for the height, the z-axis is for the width of the channel.

Further the layer of raw materials which is limited within the enclosed volume to the sizes l_1, h_1, and L is considered. The density of raw materials ρ, the speed of its motion along the axis of the channel υ_0, the tensions p_{ij} which arise in raw materials (which are the result of acts of viscous forces), the pressure P and the temperature T are considered as functions of the time t and the spatial coordinates x, y and z.

When solving problems of mechanics of continuous medium the following are unknown:

- Material density $\rho=\rho$ t, x, y, z ;
- three components of velocity vectors along the axes of the chosen coordinate system:

$$\upsilon_x=\upsilon_x \text{ t, x, y, z },\quad \upsilon_y=\upsilon_y \text{ t, x, y, z },$$
$$\upsilon_z=\upsilon_z \text{ t, x, y, z };\qquad (2)$$

- six components of strain tensor (of ten components only six are independent owing to the symmetry of the tensor $p_{ij} = p_{ji}$):

$$\sigma_{xx}=\sigma_{xx} \text{ t, x, y, z },\quad \sigma_{yy}=\sigma_{yy} \text{ t, x, y, z },$$
$$\sigma_{zz}=\sigma_{zz} \text{ t, x, y, z },\quad \tau_{xy}=\tau_{xy} \text{ t, x, y, z },$$
$$\tau_{yz}=\tau_{yz} \text{ t, x, y, z },\quad \tau_{zx}=\tau_{zx} \text{ t, x, y, z }.\qquad (3)$$

In the equations (3) the first index specifies a normal line to the platform in which this tension takes place, and the second specifies parallel to what axis this tension takes place. Thus, the normal tensions are designated as σ, and tangents as τ;

- the temperature of the supplied raw materials – $T = T$ t, x, y, z .

To define these eleven unknown values it is necessary to make and solve the system consisting of the same quantity of equations:

- the continuity equation (the mathematical expression of law of perdurability of matter according to which the weight remains constant in the closed system):

$$\frac{\partial\rho}{\partial t}+\frac{\partial}{\partial x}(\rho\cdot\upsilon_x)+\frac{\partial}{\partial y}(\rho\cdot\upsilon_y)+\frac{\partial}{\partial z}(\rho\cdot\upsilon_z)=0\quad ,\ (4)$$

- three motion equations in the chosen coordinate system (which are the mathematical formulation of Newton's second law):

$$\rho\left(\frac{\partial\upsilon_x}{\partial t}+\upsilon_x\frac{\partial\upsilon_x}{\partial x}+\upsilon_y\frac{\partial\upsilon_y}{\partial y}+\upsilon_z\frac{\partial\upsilon_z}{\partial z}\right)=\rho\cdot F_x+\frac{\partial\sigma_{xx}}{\partial x}+\frac{\partial\tau_{yx}}{\partial y}+\frac{\partial\tau_{zx}}{\partial z}\ ,$$
$$\rho\left(\frac{\partial\upsilon_y}{\partial t}+\upsilon_y\frac{\partial\upsilon_y}{\partial x}+\upsilon_y\frac{\partial\upsilon_y}{\partial y}+\upsilon_z\frac{\partial\upsilon_y}{\partial z}\right)=\rho\cdot F_y+\frac{\partial\sigma_{yy}}{\partial y}+\frac{\partial\tau_{xy}}{\partial x}+\frac{\partial\tau_{zy}}{\partial z}\ ,\ (5)$$
$$\rho\left(\frac{\partial\upsilon_z}{\partial t}+\upsilon_x\frac{\partial\upsilon_z}{\partial x}+\upsilon_y\frac{\partial\upsilon_z}{\partial y}+\upsilon_z\frac{\partial\upsilon_y}{\partial z}\right)=\rho\cdot F_z+\frac{\partial\sigma_{zz}}{\partial z}+\frac{\partial\tau_{xz}}{\partial x}+\frac{\partial\tau_{yz}}{\partial y},$$

where F_x, F_y, F_z are the projections of mass forces on to the axes of coordinates;

- six rheological equations connecting the components of tension tensor with the components of tensor of speeds of deformation of raw materials. This connection defines the rheological properties of raw materials;
- the energy equation.

To simplify the calculation when solving the system of equations the following admissible simplifications have been introduced. It is considered that an isothermal task should be considered which would provide an opportunity to do without the energy equation. The raw materials which do not compress, that is $\rho=const$, should be considered. This simplification changes the type of continuity equation - if $\rho=const$, then in that case $\frac{\partial \text{p}}{\partial \text{x}}=0$, then it follows from the continuity equation that:

$$\rho\left(\frac{\partial\upsilon_x}{\partial x}+\frac{\partial\upsilon_y}{\partial y}+\frac{\partial\upsilon_z}{\partial z}\right)=0\quad ,\qquad (6)$$

and as $\rho\neq 0$, the continuity equation takes the following form:

$$\frac{\partial\upsilon_x}{\partial x}+\frac{\partial\upsilon_y}{\partial y}+\frac{\partial\upsilon_z}{\partial z}=0\quad ,\qquad (7)$$

Analyzing the process of supply of raw materials in the top and the known rheological models of food media, we accept that the meat raw materials supplied by the top screw to its grinding knot is a viscoelastic medium the model of which consists of consistently connected bodies of Calvin and Maxwell. In that case the rheological properties of lumpy meat raw materials, according to [7], are displayed by means of the coefficient of viscoelastic properties:

$$\psi = \frac{E_\mu \cdot E_S\left(1-e^{-\frac{t_1}{t_2}}\right)}{E_S\left(1-e^{-\frac{t_1}{t_2}}\right)+t_1\cdot\eta_1+E_S\left(1-e^{-\frac{t_1}{t_2}}\right)+E_\mu}, \qquad (8)$$

where E_μ is the module of instant deformation of the body, Pa; E_S is the module of equilibrium elasticity and after-effect, Pa; t_1 is the period of relaxation of tensions, sec; t_2 is an after-effect period, sec; η_1 is viscosity, Pa · sec.

Then six rheological equations take the following form:

$$\begin{aligned}
\sigma_{xx} &= -p+2\psi\cdot\frac{\partial\upsilon_x}{\partial x}; & \tau_{xy} &= \tau_{yx} = \psi\left(\frac{\partial\upsilon_x}{\partial y}+\frac{\partial\upsilon_y}{\partial x}\right);\\
\sigma_{yy} &= -p+2\psi\cdot\frac{\partial\upsilon_y}{\partial y}; & \tau_{yz} &= \tau_{zy} = \psi\left(\frac{\partial\upsilon_y}{\partial z}+\frac{\partial\upsilon_z}{\partial y}\right);\\
\sigma_{zz} &= -p+2\psi\cdot\frac{\partial\upsilon_z}{\partial z}; & \tau_{zx} &= \tau_{xz} = \psi\left(\frac{\partial\upsilon_z}{\partial x}+\frac{\partial\upsilon_x}{\partial z}\right).
\end{aligned} \qquad (9)$$

where p is hydrostatic pressure:

$$p = -\frac{1}{3}\;\; p_{11}+p_{22}+p_{33}\;\;, \qquad (10)$$

here p_{11}, p_{22} and p_{33} are the main normal tensions.

After the substitution of the corresponding expressions in the equation (5) for the components of tension tensor from the equations (8) and the division of all of them by ρ, three equations of movement of viscous medium written down in the form of Navier-Stokes equation have been obtained:

$$\begin{aligned}
\frac{\partial\upsilon_x}{\partial t}+\upsilon_x\cdot\frac{\partial\upsilon_x}{\partial x}+\upsilon_y\cdot\frac{\partial\upsilon_x}{\partial y}+\upsilon_z\cdot\frac{\partial\upsilon_x}{\partial z} &= F_x-\frac{1}{\rho}\cdot\frac{\partial p}{\partial x}+\psi\cdot\left(\frac{\partial^2\upsilon_x}{\partial x^2}+\frac{\partial^2\upsilon_x}{\partial y^2}+\frac{\partial^2\upsilon_x}{\partial z^2}\right);\\
\frac{\partial\upsilon_y}{\partial t}+\upsilon_x\cdot\frac{\partial\upsilon_y}{\partial x}+\upsilon_y\cdot\frac{\partial\upsilon_y}{\partial y}+\upsilon_z\cdot\frac{\partial\upsilon_y}{\partial z} &= F_y-\frac{1}{\rho}\cdot\frac{\partial p}{\partial y}+\psi\cdot\left(\frac{\partial^2\upsilon_y}{\partial x^2}+\frac{\partial^2\upsilon_y}{\partial y^2}+\frac{\partial^2\upsilon_y}{\partial z^2}\right);\\
\frac{\partial\upsilon_z}{\partial t}+\upsilon_x\cdot\frac{\partial\upsilon_z}{\partial x}+\upsilon_y\cdot\frac{\partial\upsilon_z}{\partial y}+\upsilon_z\cdot\frac{\partial\upsilon_z}{\partial z} &= F_z-\frac{1}{\rho}\cdot\frac{\partial p}{\partial z}+\psi\cdot\left(\frac{\partial^2\upsilon_z}{\partial x^2}+\frac{\partial^2\upsilon_z}{\partial y^2}+\frac{\partial^2\upsilon_z}{\partial z^2}\right).
\end{aligned} \qquad (11)$$

The steady motion of material (the stationary mode) exclusively of the start-up mode and the stop of the screw are being considered. This simplification results in the equality of derivatives of time to zero. For example, $\frac{\partial\upsilon_x}{\partial t}=0$ etc. The mass forces are neglected, that is it is considered that F_x, F_y and F_z are equal to zero in the equations (10). Gravity is not considered, meaning that the movement of raw materials occurs generally by means of other sources, for example, pressure difference. The speeds of movement of raw materials in the channels are rather small which allows to neglect the inertial loadings arising in motion, as well. The laminar flow which almost always takes place when supplying with high-viscosity media is being considered. In that case, in the absence of turbulence, of three components of speed of raw materials its component on the x-axis of the channel is the only that differs from zero – υ_x, υ_y and υ_z are considered equal to zero.

Thus, for the isothermal movement of medium in case of the steady mode, the possibility to neglect mass forces and lack of turbulence Navier-Stokes equation takes the following form:

$$0=-\frac{1}{\rho}\cdot\frac{\partial p}{\partial z};\quad 0=-\frac{1}{\rho}\cdot\frac{\partial p}{\partial y};$$

$$0=-\frac{1}{\rho}\cdot\frac{\partial p}{\partial x}+\frac{\psi}{\rho}\cdot\left(\frac{\partial^2\upsilon_x}{\partial z^2}+\frac{\partial^2\upsilon_x}{\partial y^2}\right). \qquad (12)$$

The analysis of equations (11) shows that the pressure p is constant throughout the section (it does not change on the y- and z-axes) and depends only on x and changes from section to section with changes of x. In that case the partial derivative $\partial p/\partial x$ is replaced with a full derivative dp/dx in the further calculations. At the same time, assuming that $\upsilon_x=\upsilon_y=0$, $\partial\upsilon_z/\partial z=0$ has been obtained from the continuity equation (4). Therefore, unlike the pressure p the speed component υ_z does not change from section to section, and there is only the function x, y, i.e. it changes only throughout the section. Finally:

$$\upsilon_z=\upsilon_z\;\; x,y\;\;;\upsilon_z\neq\upsilon_z\;\; z\;\;;p=p\;\; z\;\;;p\neq p\;\; x,y\;\;.(13)$$

The last of the equations (11) can be written down as:

$$\frac{\partial^2\upsilon_x}{\partial z^2}+\frac{\partial^2\upsilon_x}{\partial y^2}=\frac{1}{\psi}\cdot\frac{\partial p}{\partial x}. \qquad (14)$$

If we consider that the change of pressure p along the length of the channel L is equal to Δp, one equation in the form of Poisson's equation can be obtained for the solution of problem of mechanics of continuous medium taking into account the made assumptions:

$$\frac{\partial^2 \upsilon_x}{\partial z^2} + \frac{\partial^2 \upsilon_x}{\partial y^2} = -\frac{\Delta p}{\psi l}. \qquad (15)$$

We accept that the delivery channel of the screw is rather small. Therefore we can neglect the influence of sidewalls of rounds of the screw and consider that we deal with an infinitely narrow gap for which υ_z does not change on the x-axis, that is $\frac{\partial^2 \upsilon_z}{\partial x^2} = 0$. Then Poisson's equation will become still simpler:

$$\frac{\partial^2 \upsilon_x}{\partial y^2} = -\frac{\Delta p}{\psi l} \qquad (16)$$

Using double integration on the y-axis and taking into account the boundary conditions ($\bar{\upsilon}_1 = \bar{\upsilon}_0 \neq 0;\ \bar{\upsilon}_2 = 0$), the following will be obtained:

$$\upsilon = \frac{\upsilon_0 y}{h_в} - \frac{h_в y - y^2}{2\psi} \cdot \frac{\partial p}{\partial x}.$$

On the right side of this equation the first element is the speed of direct flow of raw materials along the screw channel of the screw (with the productivity Q_{main}), and the second element is the speed which is directed in the opposite direction, i.e. the reverse flow along the screw channel (with the productivity $Q_{s.c.}$). The sum of both speeds is the speed of net flow.

It follows from the equation (16) that if we integrate the speeds according to the section of the flow, in other words throughout the height h_1 and the width l_1, then the intensity of net flow (the amount of weight which passes along the screw per unit of time) will be stated for the bilamellate model using the formula:

$$Q_{main} = \frac{\upsilon_0 l_1 h_1}{2} - \frac{l_1 h_1^3}{12\psi} \cdot \frac{\partial p}{\partial x}. \qquad (17)$$

The first element in the right part of the formula (17) is the direct flow Q_{main}, and the second is the reverse flow $Q_{s.c.}$. At the same time it should be noted that the influence of braking action of sidewalls of the pitch between the rounds of the screw on the flow is not taken into consideration.

Let us apply a change for the parameters which are part of the equation (17):

$$\begin{cases} \upsilon_0 = \pi \cdot D \cdot n \cdot \cos\alpha; \\ l_1 = \;\; \pi \cdot D \cdot tg\alpha - l_1 \;\; \cdot \cos\alpha = \pi \cdot D \cdot \sin\alpha - l_1 \cdot \cos\alpha; \\ dz = dl \cdot \sin\alpha; \end{cases} \qquad (18)$$

where α is the angle of lead of rounds of the screw.

Then the statement (17) of definition of intensity of net flow (without losses through the gap $Q_{i.g.}$) will be written as:

$$Q_{main} = \frac{\pi \cdot D \cdot n \cdot \cos\alpha \cdot \;\; \pi \cdot D \cdot \sin\alpha - l_1 \cdot \cos\alpha \;\; h_1}{2} - \frac{\sin\alpha \cdot \;\; \pi \cdot D \cdot \sin\alpha - l_1 \cdot \cos\alpha \;\; h_1^3}{12\psi} \cdot \frac{\partial p}{\partial l} \qquad (19)$$

Thus, the reverse flow is proportional to the hollow width, and also to the third degree of depth of round and is inversely proportional to the length of rectangular channel between the rounds.

We will obtain the statement of definition of intensity of flow of losses $Q_{i.g.}$through the gap between the screw and the working cylinder. This flow is considered as the one that passes through a rectangular slot with the sides $\pi \cdot D/\cos\alpha$, δ and $b \cdot \cos\alpha$. This flow follows the equation (15) with the corresponding boundary conditions. At the same time it is necessary to take into account that the reverse flow $Q_{i.g.}$in the gap is caused by a considerably greater pressure difference than the reverse flow $Q_{s.c.}$ which is there in the screw channel. The values of pressure differences are related as the length of a pitch of the screw around to the thickness of a round.

Taking this into account:

$$\frac{\pi \cdot D}{\cos\alpha} b \cdot \cos\alpha = \frac{\pi \cdot D}{l_1 \cdot \cos^2\alpha}. \qquad (20)$$

Then we will obtain the statement similar to the statement of definition of reverse flow through the screw channel $Q_{s.c.}$. The following replacements are only needed:

$$\begin{cases} \pi \cdot D \cdot \sin\alpha \rightarrow \frac{\pi \cdot D}{\cos\alpha}; \\ h_1 \rightarrow \delta; \\ \frac{dl}{\sin\alpha} \rightarrow b \cdot \cos\alpha; \\ \frac{dp}{dl} \rightarrow \pi \cdot D \cdot \sin\alpha \cdot \cos\alpha \cdot \frac{dp}{dl}. \end{cases} \qquad (21)$$

It thus appears that we have the following:

$$Q_{s.c.} = \frac{\pi^2 \cdot D^2 \cdot \delta^2 \cdot tg\alpha}{12\psi \cdot b} \cdot \frac{\partial p}{\partial l}. \qquad (22)$$

For the purpose of practical application it is expedient to change a little the equations (19, 22) introducing, instead of the local pressure gradient *dp/dl* which is accepted constant throughout the length of the way of flow of mass of raw materials, the following value:

$$\frac{dp}{dl} = \frac{p_2 - p_1}{L_{1,2}}. \qquad (23)$$

where p_1 is the weight pressure in the beginning of supply section; p_2 is the weight pressure in the end of supply section; $L_{1,2}$ is the length of supply section.

Thus, the statements for the definition of separate components of net flow will take the following form:

- the direct flow along the screw type pump:

$$Q_{main}=\frac{\pi^2\cdot D^2\cdot n\cdot h_1\cdot\sin\alpha\cdot\cos\alpha}{2};\qquad(24)$$

- the reverse flow along the screw channel:

$$Q_{s.c.}=\frac{\pi\cdot D\cdot h_1^3\cdot\sin^2\alpha}{12\psi}\cdot\frac{p_2-p_1}{L_{1,2}};\qquad(25)$$

- the reverse flow in the gap between the screw and the cylinder:

$$Q_{i.g.}=\frac{\pi^2\cdot D^2\cdot\delta^2}{12\psi\cdot b}\cdot tg\alpha\cdot\frac{p_2-p_1}{L_{1,2}}.\qquad(26)$$

The statement, as for the determination of coefficient of productivity of the top $K_{Q,}$ has the form [6], the scheme of its definition is provided in Fig. 2:

$$K_Q=\frac{\beta_{\max}}{360}-\frac{4S_b^{act}\cdot z_b^{act}}{\varphi\cdot\pi\ \ D_g^2-d_g^2}=1-\frac{4S_b^{act}\cdot z_b^{act}}{\varphi\cdot\pi\ \ D_g^2-d_g^2}-\left(\theta_{pen}\cdot\left(\frac{1-\varphi}{\varphi}\right)+\theta_{cut}+\left(\frac{q_0}{k_l}\right)\cdot e^{\frac{4f\cdot k_l\cdot B_g}{d_0}}-\frac{q_0}{k_l}\right)\times$$
$$\times\left(\frac{\varphi\cdot k_{\text{int}}\cdot H_{o-1}}{E\cdot N_{fil.}}\right)\left(\frac{\pi\cdot n\cdot e^{a_\upsilon}\ \ D-h_1\ \ \sin\alpha}{\cos\gamma_{mp}}\cdot\cos\ \ \alpha+\gamma_{fr}\ \ \right)^{b_\upsilon},\qquad(27)$$

where D_g is the outer diameter of the grating; d_g is the diameter of central hole of the grating; S_b^{act} is the area of the front projection of knife blade which is in the section of supply of raw materials contoured by the angle β_{max}; z_b^{act} is the quantity of knife blades which are in the section of supply of raw materials contoured by the angle β_{max}; k_{int} is the empirical coefficient of increase in the resistance of grinding knot due to an increase in the distances between separate gratings by the values of thickness of knives the couples of grinders with gratings are composed of; φ is the coefficient of utilization of working area of the initial grating; θ_{pen} is the tension of penetration of raw materials when flowing around the jumpers between the holes of the grating, Pa; θ_{cut} is the tension of cutoff of raw materials during indention into the holes of the grating, Pa; E is the modulus of elasticity of raw materials during compression, Pa; f is the coefficient of friction of raw materials on the walls of the channel; B_g is the thickness of the grating, m; d_h is the diameter of holes of the grating, m; q_0 is the residual lateral pressure, Pa; k_l is the coefficient of lateral pressure; a_υ and b_υ are the empirical coefficients of increase in the resistance of grinding knot due to an increase in the speed of supply of raw materials the value of which depends on the lengthening of the holes of the grating and the type of raw materials; γ_{fr} is the angle of friction of raw materials on the surface of the screw; h_1 is the depth of the screw channel of the screw; n_s is the rotation frequency of the screw; H_{o-1} is a pitch between the rounds of the screw, within the last round; $N_{fil.}$is a pitch between the rounds of the screw when the coefficient of its filling with raw materials is equal to 1; α is the angle of lead of rounds of the screw.

The experimental studies of accuracy of the mathematical model were performed in the sausage shop of the meat-processing enterprise LLC Cherkasskaya prodovol'stvennaya kompaniya. The objects of researches were the tops of the models K6-FVZP-200, VVS-180, MP-160 and AL-130 with the gratings with the outer diameter of respectively 200 mm, 180 mm, 160 mm and 130 mm. Fig. 3 provides the external view of the tops during the researches performed by the authors of the work.

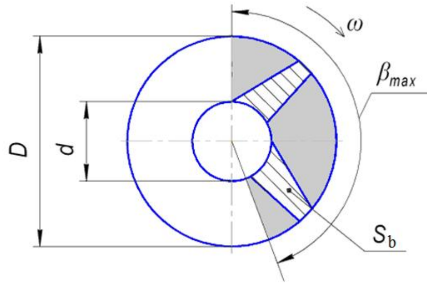


Fig. 2. Design scheme of the last round of the screw (the front view) when determining the coefficient of productivity of the top and K_Q.

RESULTS AND DISCUSSION

According to the obtained results (Fig. 4), the above stated structural and kinematic parameters essentially influence the productivity of the top.

The increase in the outer diameter of grating D_g provides an increase in productivity due to the growth of quantity of holes for passing of raw materials (Fig. 4, a). Thus, with an increase in the diameter of the grating from 130 mm to 170 mm the productivity increases by 2.3 times.

The increase in the angle of lead of rounds of the screw α from 2° to 5° provides an increase in productivity up to 2 times (Fig. 4, b). At the same time the further increase α provides the reduction of productivity of the top. It can be explained by the fact that the screw with a high value α provides the movement of a portion of raw materials to a greater axial distance per one turn (in that case the productivity of the top increases). But at the same time the percentage of force which moves the raw materials in the axial direction of the screw decreases. In the presence of hydraulic resistance from the grinding knot such reduction of force of supply provides the fact that a lower amount of raw materials is pressed through the holes of the grating, the raw materials begin to slip on the rounds of the screw, to move back along the screw channel and through the gap between the screw and the cylinder. As a result, with low values of hydraulic resistance of the grinding knot (with the reduced values of B_p) the increase in α provides an increase in the productivity of the top (Fig. 4, c), and with the increased hydraulic resistance of the grinding knot (with the increased values of B_g) the increase in α provides a decrease in the production of the top.

For the tops which are designed for operation with the output gratings having small holes (the grinding of raw materials to the state of forcemeat) screws with $\alpha = 4–5^{\circ}$ can be recommended, and for the tops which are designed for operation with the output gratings having large holes (the grinding of raw materials to the state of meal) screws with $\alpha = 5.5–8^{\circ}$ can be recommended.

The influence of rotation frequency of the screw n (Fig. 4, d) is similar to the influence of the angle of lead of rounds of the screw - with an increase in frequency the productivity first increases, reaches the maximum and then decreases. The maximum values of productivity are observed at n=4–6 sec^{-1}. The further increase in n reduces productivity. It can be explained by the fact that to a higher speed of movement of raw materials through the holes of grating their higher hydraulic resistance corresponds. And it predetermines, among other things, an increase in the amount of raw materials which move between the rounds of the screw and through the gap between the screw and the cylinder backwards from the grinding knot which provides the reduction of productivity.

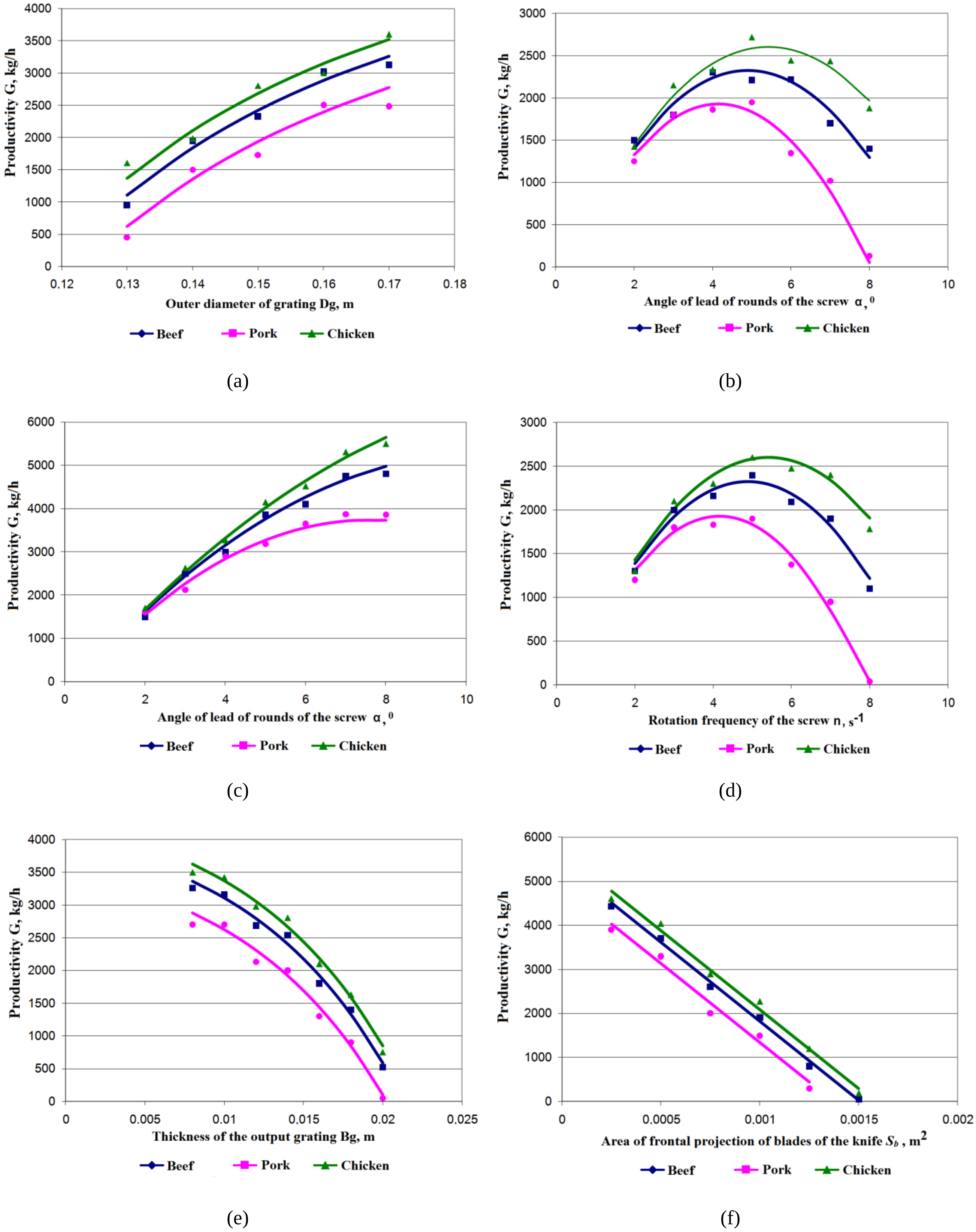
The increase in the thickness of the output grating B_g provides the rapid reduction of productivity (Fig. 4, e). It is explained by the growth of hydraulic resistance of holes with an increase in their length (hydraulic resistance grows under the exponential law). As a result, wuth an increase in the thickness of the output grating from 8 mm to 20 mm the productivity decreases, depending on the type of raw materials, by 4.5–6.8 times.

The dependence of productivity of the top on the area S_b of the frontal projection of blades of the knife which operates together with the output grating is inversely proportional (Fig. 4, f). In this case the reduction of productivity of the top with an increase in the total area of blades occurs due to the overlapping of a higher quantity of holes for the passing of raw materials.

(a) (b)

(c) (d)

Fig. 3. Tops under study: (a) К6-FVZP-200, (b) VVS-180, (c) MP-160, (d) AL-130.



(a) (b) (c) (d) (e) (f)

Fig. 4. Dependence of the productivity of the meat grinder on its structural and kinematic parameters: (a) the outer diameter of grating; (b) the angle of lead of rounds of the screw, $B_g=0.016$ *m*; (c) the angle of lead of rounds of the screw, $B_g=0.008$ *m*; (d) the rotation frequency of the screw; (e) the thickness of the output grating; (f) the area of frontal projection of blades of the knife.

The performed studies have confirmed the results of mathematical modeling. The curves drawn according to the mathematical model are identical to the experimental ones, the error does not exceed 11%.

The productivity G, kg/h, was determined when revealing the rational parameters of process of grinding using the parameter of optimization:

$$G = f(D_g, n, \alpha, S_b, B_g), \tag{28}$$

where D_g is the outer diameter of the output grating of the grinding knot; n is the rotation frequency of the screw; α is the angle of lead of rounds of the screw; S_b is the area of frontal projection of one knife blade; B_g is the thickness of the initial grating of the grinding knot.

Due to the significant amount of factors in this case an expediently statistical analysis has been performed to obtain functional dependence in the form of multiple regression of the second order by means of central composite rotatable design (CCRD) of multiple-factor experiment. The method of CCRD allows to obtain more precisely the mathematical description of data distribution due to an increase in the number of experiments in the central points of plan matrix and the special choice of "star value".

The analysis of statistical characteristics of the obtained data has shown that the coefficients of their asymmetry tend to zero, that is the distribution of experimental data is symmetric and is approximated according to a normal law. The choice of ranges of variation of factors of functions was made in the way that their any combination provided by the plan of experiment could be realized within these intervals and did not provide contradictions. For this purpose searching experiments have been made to determine the areas in which the necessary combinations of levels of factors would be steadily realized. All the factors that are part of a searching function are the values that have different dimensions, and the values of these factors have various orders. Therefore, to obtain a response surface of these functions an operation of coding of factors has been performed, which is the linear transformation of factorial space.

It has been planned to obtain a multiple regression equation of the 2nd order:

$$y = b_0 + \sum_{i=1}^{n} b_i x_i + \sum_{i=1}^{n} b_{ij} x_i^2 + \sum_{i=1}^{n} b_{ij} x_{ij} x_{ij}, \tag{29}$$

where y is one of high-quality functions; b_{ij} are the regression coefficients obtained using the method of the smallest squares.

For the assessment of adequacy of the obtained regression equation analytical and graphic methods of analysis have been used. The hypothesis of reproducibility of experiences is proved by means of Cochran's criterion which shows that at the level of confidential probability of 95% dispersions are uniform because the design value of criterion is less than the tabular one. The assessment of importance of regression coefficients has been performed using Student criterion t. The assessment of adequacy of the obtained mathematical model has been performed using Fisher criterion which has shown that the design values are much lower than the critical ones, therefore the obtained regression model adequately describes the response surface, and it can be used for the optimization of the studied processes.

The analysis of distribution of the initial residues based on the predicted values has shown that they have a chaotic character of arrangement on the surface and there is no regularity in their behavior. Based on these observations, it is possible to draw a conclusion that the residues have no correlation relationships among themselves, that is the regression model sufficiently describes the interrelation of experimental values and is adequate.

Based on the results of the made experiments, studies and tests of the developed equipment for the process of grinding of meat raw materials, optimum technological parameters of its work (Table 1) the compromise value of which has been obtained using Cramer's method in the mathematical environment "MathCAD 15" have been determined (Fig. 5–7).

As a result of statistical optimization a correlation and regression equation of functional dependence of productivity of the process of supply of raw materials on constructive technology factors has been obtained. After processing of experimental data in the statistical environment STATISTICA 10.0 coefficients of complex equation of multiple regression of the 2nd order were obtained and the following dependence was derived:

$$\begin{gathered} G = -80127 + 702987 D_g + 3249 n + 3256\alpha + 1224813 S_b + 2151062 B_g - 2550000 D_g^{\,2} - \\ -367 n^{2} - 349\alpha^2 - 87187500 S_b^{\,2} - 88750000 B_g^{\,2} + 4750 D_g n + 5125 D_p \cdot \alpha + 2468750 D_g \cdot S_b - \\ -812500 D_g B_g + 4062 n \cdot S_b - 33125 n \cdot B_g - 19063\alpha \cdot S_b - 36250\alpha \cdot B_b \end{gathered} \tag{30}$$

Table 1. Optimum technological parameters of the studied grinding process

Technological parameter	Rational value
Outer diameter of the output grating of the grinding knot, m	0.15–0.155
Rotation frequency of the screw, sec^{-1}	4.5–5.2
Angle of lead of rounds of the screw, degrees	4.8–5.5
Area of the frontal projection of one knife blade, m^2	0.001–0.0011
Thickness of the initial grating of the grinding knot, m	0.0075–0.0082

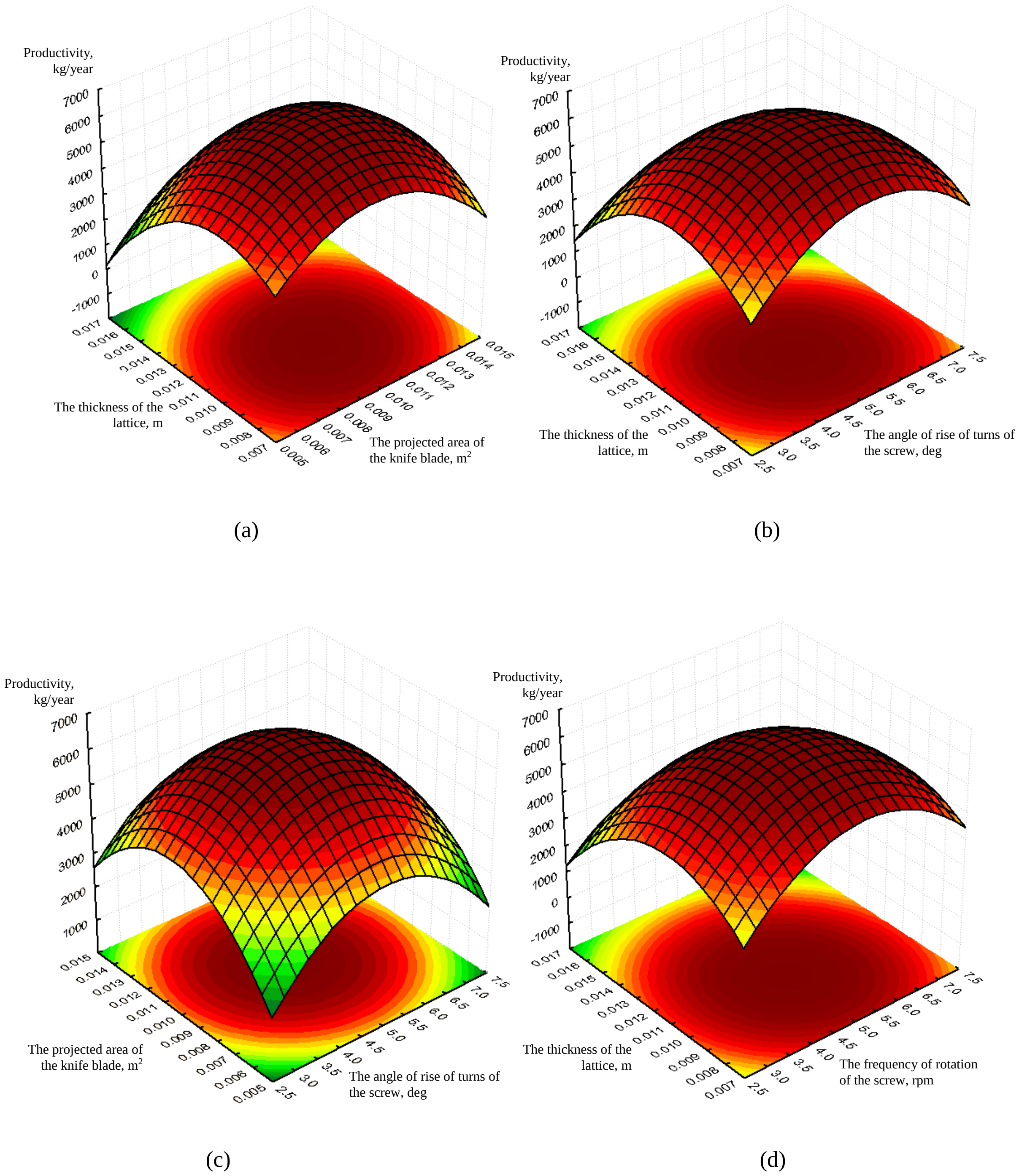
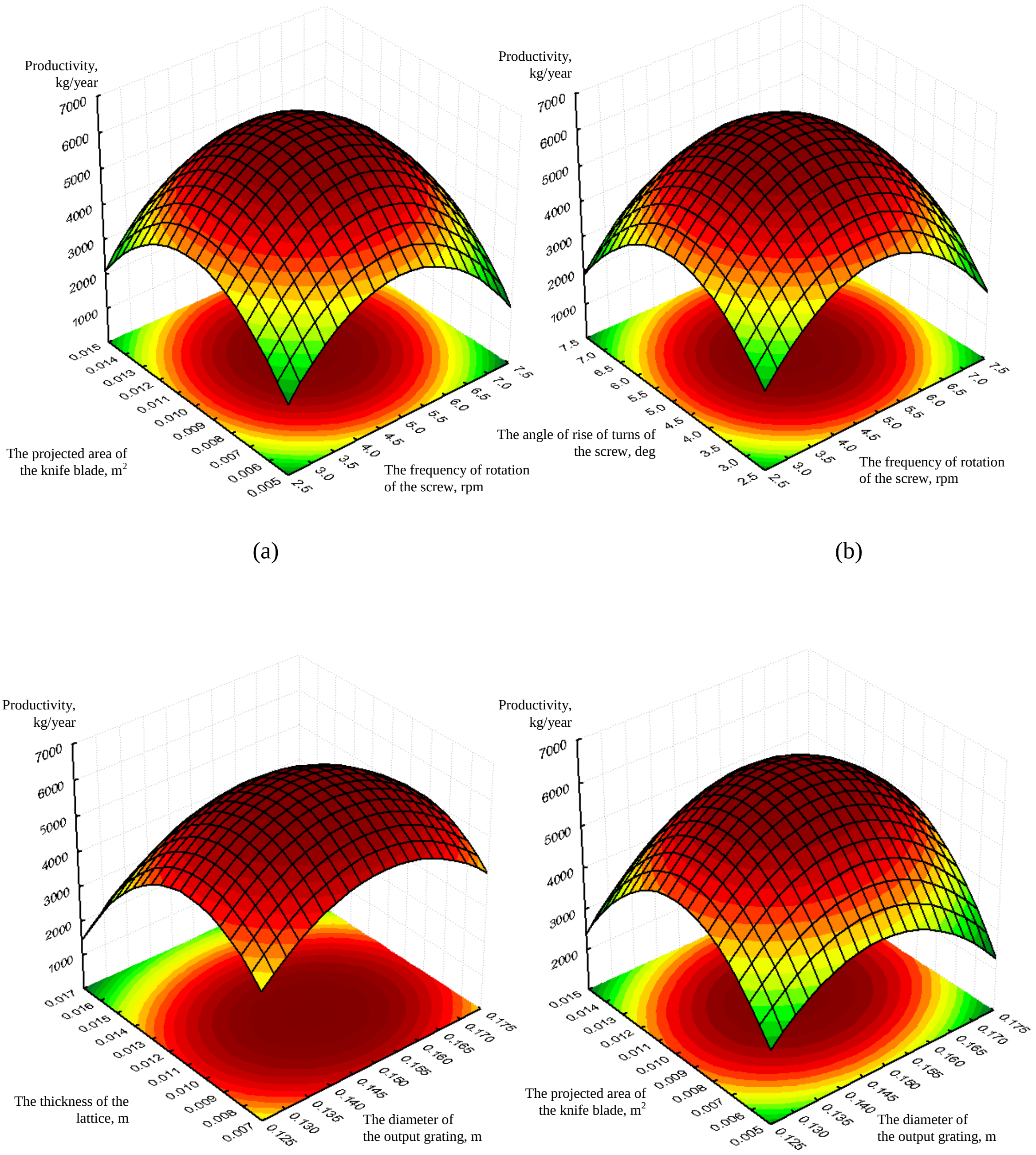


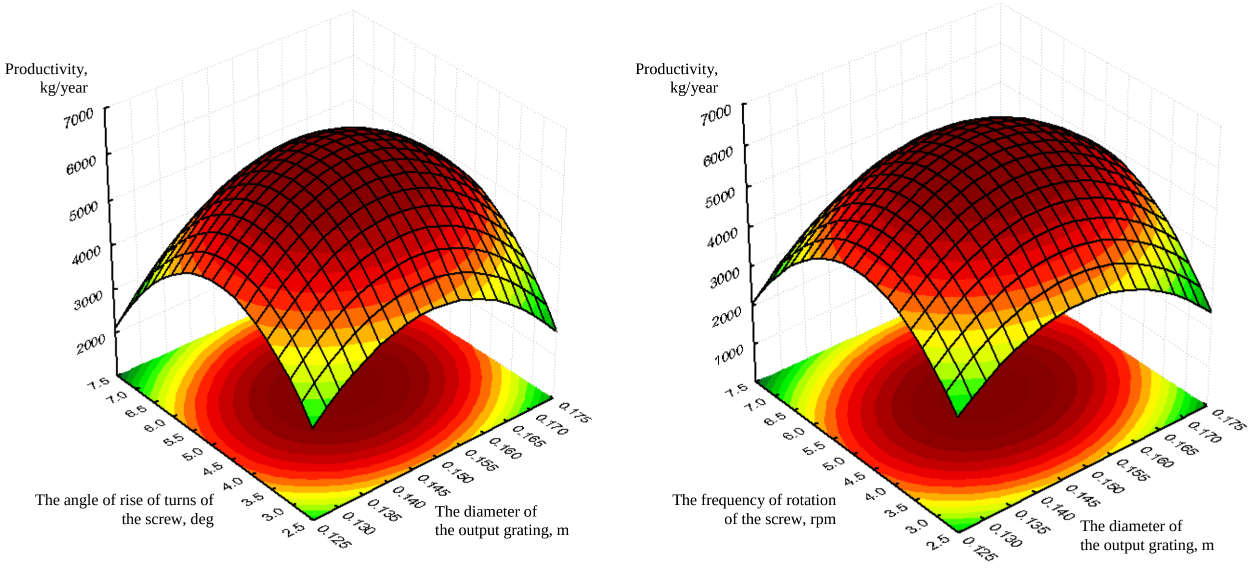
Fig. 5. Surfaces of responses and their projections for productivity in the interaction of major factors: (a) the area of the front projection of one knife blade and the thickness of the initial grating; (b) the thickness of the initial grating and the angle of lead of rounds of the screw; (c) the area of the front projection of one knife blade and the angle of lead of rounds of the screw; (d) the thickness of the initial grating and the rotation frequency of the screw.



(a) (b)

(c) (d)

Fig. 6. Surfaces of responses and their projections for productivity in the interaction of major factors: (a) the area of the front projection of one knife blade and the rotation frequency of the screw; (b) the angle of lead of rounds of the screw and the rotation frequency of the screw; (c) the thickness of the initial grating and its outer diameter; (d) the area of the front projection of one knife blade and the outer diameter of the initial grating of the grinding knot.



(a) (b)

Fig. 7. Response surfaces and their projections for productivity in the interaction of major factors: (a) the angle of lead of rounds of the screw and the diameter of the output grating of the grinding knot; (b) the rotation frequency of the screw and the diameter of the output grating of the grinding knot.

As a result of the work, a mathematical model of the process of supply of raw materials in the top has been developed that allows to increase significantly the accuracy of determination of its productivity and also to set the most rational structural and kinematic parameters of the working bodies of the top.

By means of the obtained mathematical model the influence of structural and kinematic parameters of the top on its productivity has been studied. The model considers the phenomenon of supply of meat raw materials only in a certain sector of the last round of the screw of the top. It has been established that the productivity of the top is defined, first of all, by the supplying ability of the screw which forms the main flow of raw materials through the grinding knot. The presence of gratings and knives in the grinding knot and their corresponding geometrical parameters provide the formation of hydraulic resistance of the grinding knot which prevents from the free effluence of raw materials from the working cylinder under the influence of the pressure produced by the screw. Thereof the reverse flows of raw materials are formed characterizing losses of productivity - through the gap between the external surface of rounds of the screw and the internal surface of the working cylinder and along the screw channel of the screw. An essential increase in the productivity of the top can be reached increasing the rotation frequency of the screw, increasing its outer diameter, reducing the depth of a round, reducing the gap between the screw and the working cylinder, and also increasing the thickness of a screw round.

The following optimum technological parameters of the process of supply of raw materials in the top have been determined as a result of statistical optimization: the outer diameter of the output grating is 0.15–0.155 m; the rotation frequency of the screw is 4.5–5.2 sec^{-1}; the angle of lead of rounds of the screw is 4.8–5.5 degrees; the area of frontal projection of a knife blade is 0.001–0.0011 m^2; the thickness of the output grating is 0.0075–0.0082 m.

The use of the obtained mathematical model allows to increase the accuracy of design calculations of tops and to prove new, highly productive, ways of supply of raw materials to the grinding knot of tops.

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